

# Deep Learning

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# 1 Neural networks expressivity

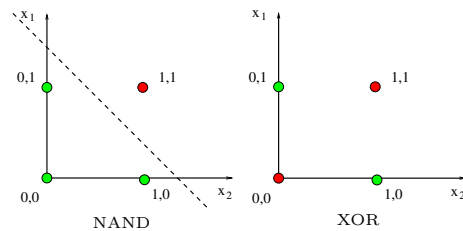
## 1.1 Perceptron

Single neuron that defines a binary threshold through a hyperplane:

$$\begin{cases} 1 & \sum_i w_i x_i + b \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

**Expressivity** A perceptron can represent a NAND gate but not a XOR gate.

Perceptron  
expressivity



**Remark.** Even if NAND is logically complete, the strict definition of a perceptron is not a composition of them.

## 1.2 Multi-layer perceptron

Composition of perceptrons.

**Shallow neural network** Neural network with one hidden layer.

Shallow NN

**Deep neural network** Neural network with more than one hidden layer.

Deep NN

**Expressivity** Shallow neural networks allow to approximate any continuous function

Multi-layer  
perceptron  
expressivity

$$f : \mathbb{R} \rightarrow [0, 1]$$

**Remark.** Still, deep neural networks allow to use less neural units.