Fundamentals of Artificial Intelligence and Knowledge Representation (Module 2)

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Contents

1 Propositional and first order logic

See `Languages and Algorithms for AI (module 2)`.

2 Ontologies

Ontology Formal (non-ambiguous) and explicit (obtainable through a finite sound procedure) description of a domain. Ontology

Category Can be organized hierarchically on different levels of generality. Category

Object Belongs to one or more categories. Object

Upper/general ontology Ontology focused on the most general domain. Upper/general ontology

Properties:

- Should be applicable to almost any special domain.
- Combining general concepts should not incur in inconsistences.

Approaches to create ontologies:

- Created by philosophers/logicians/researchers.
- Automatic knowledge extraction from well-structured databases.
- Created from text documents (e.g. web).
- Crowd-sharing information.

2.1 Categories

Category Used in human reasoning when the goal is category-driven (in contrast to specific-instance-driven). Category

In first order logic, categories can be represented through:

Predicate A predicate to tell if an object belongs to a category (e.g. `Car(c1)` indicates that `c1` is a car). Predicate categories

Reification Represent categories as objects as well (e.g. $\texttt{c1} \in \texttt{Car}$). Reification

2.1.1 Reification properties and operations

Membership Indicates if an object belongs to a category. (e.g. $\texttt{c1} \in \texttt{Car}$). Membership

Subclass Indicates if a category is a subcategory of another one. (e.g. $\texttt{Car} \subset \texttt{Vehicle}$). Subclass

Necessity Members of a category enjoy some properties (e.g. $(\texttt{x} \in \texttt{Car}) \Rightarrow \texttt{hasWheels(x)}$). Necessity

Sufficiency Sufficient conditions to be part of a category (e.g. $\texttt{hasPlate(x)} \land \texttt{hasWheels(x)} \Rightarrow \texttt{x} \in \texttt{Car}$). Sufficiency

Category-level properties Category themselves can enjoy properties (e.g. $\texttt{Car} \in \texttt{VehicleType}$) Category-level properties

Disjointness Given a set of categories S, the categories in S are disjoint iff they all have different objects: Disjointness

$$\texttt{disjoint}(S) \iff (\forall c_1, c_2 \in S, c_1 \neq c_2 \Rightarrow c_1 \cap c_2 = \emptyset)$$

Exhaustive decomposition Given a category c and a set of categories S, S is an exhaustive decomposition of c iff any element in c belongs to at least a category in S: Exhaustive decomposition

$$\texttt{exhaustiveDecomposition}(S,\ c) \iff (\forall o \in c \iff \exists c_2 \in S : o \in c_2)$$

Partition Given a category c and a set of categories S, S is a partition of c when: Partition

$$\texttt{partition}(S,\ c) \iff \texttt{disjoint}(S) \land \texttt{exhaustiveDecomposition}(S,\ c)$$

2.1.2 Physical composition

Objects (meronyms) are part of a whole (holonym).

Part-of If the objects have a structural relation (e.g. `partOf(cylinder1, engine1)`). Part-of

Properties:

Transitivity $\texttt{partOf(x, y)} \land \texttt{partOf(y, z)} \Rightarrow \texttt{partOf(x, z)}$

Reflexivity `partOf(x, x)`

Bunch-of If the objects do not have a structural relation. Useful to define a composition of countable objects (e.g. `bunchOf(nail1, nail3, nail4)`). Bunch-of

2.1.3 Measures

A property of objects.

Quantitative measure Something that can be measured using a unit (e.g. $\texttt{length(table1)} = \texttt{cm(80)}$). Quantitative measure

Qualitative measures propagate when using `partOf` or `bunchOf` (e.g. the weight of a car is the sum of its parts).

Qualitative measure Something that can be measured using terms with a partial or total order relation (e.g. $\{\texttt{good}, \texttt{neutral}, \texttt{bad}\}$). Qualitative measure

Qualitative measures do not propagate when using `partOf` or `bunchOf`.

Fuzzy logic Provides a semantics to qualitative measures (i.e. convert qualitative to quantitative). Fuzzy logic

2.1.4 Things vs stuff

Intrinsic property Related to the substance of the object. It is retained when the object is divided (e.g. water boils at 100°C). Intrinsic property

Extrinsic property Related to the structure of the object. It is not retained when the object is divided (e.g. the weight of an object changes when split). Extrinsic property

Substance Category of objects with only intrinsic properties. Substance

Stuff The most general substance category. Stuff

Count noun Category of objects with only extrinsic properties. Count noun

Things The most general object category. Things

2.2 Semantic networks

Graphical representation of objects and categories connected through labelled links.

Semantic networks

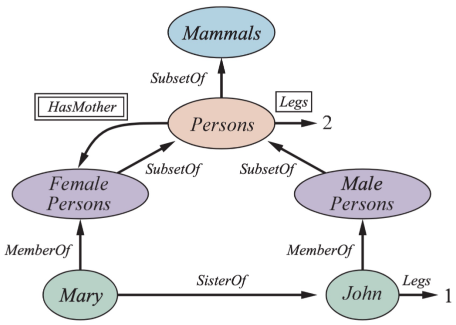


Figure 2.1: Example of semantic network

Objects and categories Represented using the same symbol.

Links Four different types of links:

- Relation between objects (e.g. `SisterOf`).
- Property of a category (e.g. 2 `Legs`).
- Is-a relation (e.g. `SubsetOf`).
- Property of the members of a category (e.g. `HasMother`).

Single inheritance reasoning Starting from an object, check if it has the queried property. If not, iteratively move up to the category it belongs to and check for the property.

Single inheritance reasoning

Multiple inheritance reasoning Reasoning is not possible as it is not clear which parent to choose.

Multiple inheritance reasoning

Limitations Compared to first order logic, semantic networks do not have:

- Negations.
- Universally and existentially quantified properties.
- Disjunctions.
- Nested function symbols.

Many semantic network systems allow to attach special procedures to handle special cases that the standard inference algorithm cannot handle. This approach is powerful but does not have a corresponding logical meaning.

Advantages With semantic networks it is easy to attach default properties to categories and override them on the objects (i.e. `Legs` of `John`).

2.3 Frames

Knowledge that describes an object in terms of its properties. Each frame has:

Frames

- An unique name
- Properties represented as pairs <slot - filler>

Example.

```
(
    toronto
        <:Instance-Of City>
        <:Province ontario>
        <:Population 4.5M>
)
```

Prototype Members of a category used as comparison metric to determine if another object belongs to the same class (i.e. an object belongs to a category if it is similar enough to the prototypes of that category). Prototype

Defeasible value Value that is allowed to be different when comparing an object to a prototype. Defeasible value

Facets Additional information contained in a slot for its filler (e.g. default value, type, domain). Facets

Procedural information Fillers can be a procedure that can be activated by specific facets:

`if-needed` Looks for the value of the slot.

`if-added` Adds a value.

`if-removed` Removes a value.

Example.

```
(
    toronto
        <:Instance-Of City>
        <:Province ontario>
        <:Population [if-needed QueryDB]>
)
```

3 Description logic

3.1 Syntax

Logical symbols Symbols with fixed meaning. Logical symbols

Punctuation ( ) [ ]

Positive integers

Concept-forming operators `ALL`, `EXISTS`, `FILLS`, `AND`

Connectives $\sqsubseteq$, $\doteq$, $\rightarrow$

Non-logical symbols Domain-dependant symbols. Non-logical symbols

Atomic concepts Categories (CamelCase, e.g. `Person`).

Roles Used to describe objects (:CamelCase, e.g. `:Height`).

Constants (camelCase, e.g. `johnDoe`).

Complex concept Concept-forming operators can be used to combine atomic concepts and form complex concepts. A well-formed concept follows the conditions: Complex concept

- An atomic concept is a concept.
- If `r` is a role and `d` is a concept, then `[ALL r d]` is a concept.
- If `r` is a role and n is a positive integer, then `[EXISTS` n `r]` is a concept.
- If `r` is a role and `c` is a constant, then `[FILLS r c]` is a concept.
- If $\mathtt{d}_1 \dots \mathtt{d}_n$ are concepts, then `[AND` $\mathtt{d}_1 \dots \mathtt{d}_n$`]` is a concept.

Sentence Connectives can be used to combine concepts and form sentences. A well-formed sentence follows the conditions: Sentence

- If $\mathtt{d}_1$ and $\mathtt{d}_2$ are concepts, then $(\mathtt{d}_1 \sqsubseteq \mathtt{d}_2)$ is a sentence.
- If $\mathtt{d}_1$ and $\mathtt{d}_2$ are concepts, then $(\mathtt{d}_1 \doteq \mathtt{d}_2)$ is a sentence.
- If `c` is a constant and `d` is a concept, then $(\mathtt{c} \rightarrow \mathtt{d})$ is a sentence.

Knowledge base Collection of sentences. Knowledge base

Constants are individuals of the domain.

Concepts are categories of individuals.

Roles are binary relations between individuals.

Assetion box (A-box) List of facts about individuals. Assetion box (A-box)

Terminological box (T-box) List of sentences (axioms) about concepts. Terminological box (T-box)

3.2 Semantics

3.2.1 Concept-forming operators

Concept-forming operators

Let r be a role, d be a concept, c be a constant and n a positive integer. The semantics of concept-forming operators are:

[ALL r d] Individuals r-related to the individuals of the category d.

Example. [ALL :HasChild Male] individuals that have zero children or only male children.

[EXISTS n r] Individuals r-related to at least n other individuals.

Example. [EXISTS 1 :Child] individuals with at least one child.

[FILLS r c] Individuals r-related to the individual c.

Example. [FILLS :Child john] individuals with child john.

[AND $d_1 \dots d_n$] Individuals belonging to all the categories $d_1 \dots d_n$.

3.2.2 Sentences

Sentences

Sentences are expressions with truth values in the domain. Let d be a concept and c be a constant. The semantics of sentences are:

$d_1 \sqsubseteq d_2$ Concept d_1 is subsumed by d_2.

Example. PhDStudent $\sqsubseteq$ Student as every PhD is also a student.

$d_1 \doteq d_2$ Concept d_1 is equivalent to d_2.

Example. PhDStudent $\doteq$ [AND Student :Graduated :HasFunding]

$c \rightarrow d$ The individual c satisfies the description of the concept d.

Example. federico $\rightarrow$ Professor

3.2.3 Interpretation

Interpretation

Interpretation An interpretation $\mathfrak{I}$ in description logic is a pair $(\mathcal{D}, \mathcal{I})$ where:

- $\mathcal{D}$ is the domain.
- $\mathcal{I}$ is the interpretation mapping.

  Constant Let c be a constant, $\mathcal{I}[\mathtt{c}] \in \mathcal{D}$.

  Atomic concept Let a be an atomic concept, $\mathcal{I}[\mathtt{a}] \subseteq \mathcal{D}$.

  Role Let r be a role, $\mathcal{I}[\mathtt{r}] \subseteq \mathcal{D} \times \mathcal{D}$.

  Thing The concept Thing corresponds to the domain: $\mathcal{I}[\mathtt{Thing}] = \mathcal{D}$.

  [ALL r d]

$$\mathcal{I}[\texttt{[ALL r d]}] = \{\mathtt{x} \in \mathcal{D} \mid \forall \mathtt{y} : \langle \mathtt{x}, \mathtt{y} \rangle \in \mathcal{I}[r] \text{ then } \mathtt{y} \in \mathcal{I}[d]\}$$

  [EXISTS n r]

$$\mathcal{I}[\texttt{[EXISTS } n \texttt{ r]}] = \{\mathtt{x} \in \mathcal{D} \mid \text{ exists at least } n \text{ distinct } \mathtt{y} : \langle \mathtt{x}, \mathtt{y} \rangle \in \mathcal{I}[r]\}$$

[FILLS r c]

$$\mathcal{I}[\texttt{[FILLS r c]}] = \{\texttt{x} \in \mathcal{D} \mid \langle \texttt{x}, \mathcal{I}[\texttt{c}] \rangle \in \mathcal{I}[\texttt{r}]\}$$

[AND $\texttt{d}_1 \dots \texttt{d}_n$]

$$\mathcal{I}[\texttt{[AND } \texttt{d}_1 \dots \texttt{d}_n\texttt{]}] = \mathcal{I}[\texttt{d}_1] \cap \dots \cap \mathcal{I}[\texttt{d}_n]$$

Model Given an interpretation $\mathfrak{I} = (\mathcal{D}, \mathcal{I})$, a sentence is true under $\mathfrak{I}$ ($\mathfrak{I} \models$ sentence) if: Model

- $\mathfrak{I} \models (\texttt{c} \rightarrow \texttt{d})$ iff $\mathcal{I}[\texttt{c}] \in \mathcal{I}[\texttt{d}]$.
- $\mathfrak{I} \models (\texttt{d}_1 \sqsubseteq \texttt{d}_2)$ iff $\mathcal{I}[\texttt{d}_1] \subseteq \mathcal{I}[\texttt{d}_2]$.
- $\mathfrak{I} \models (\texttt{d}_1 \doteq \texttt{d}_2)$ iff $\mathcal{I}[\texttt{d}_1] = \mathcal{I}[\texttt{d}_2]$.

Given a set of sentences S, $\mathfrak{I}$ models S if $\mathfrak{I} \models S$.

Entailment A set of sentences S logically entails a sentence α if: Entailment

$$\forall \mathfrak{I} : (\mathfrak{I} \models S) \rightarrow (\mathfrak{I} \models \alpha)$$

3.3 Reasoning

3.3.1 T-box reasoning

Given a knowledge base of a set of sentences S, we would like to be able to determine the following:

Satisfiability A concept $\texttt{d}$ is satisfiable w.r.t. S if: Satisfiability

$$\exists \mathfrak{I}, (\mathfrak{I} \models S) : \mathfrak{I}[\texttt{d}] \neq \varnothing$$

Subsumption A concept $\texttt{d}_1$ is subsumed by $\texttt{d}_2$ w.r.t. S if: Subsumption

$$\forall \mathfrak{I}, (\mathfrak{I} \models S) : \mathfrak{I}[\texttt{d}_1] \subseteq \mathfrak{I}[\texttt{d}_2]$$

Equivalence A concept $\texttt{d}_1$ is equivalent to $\texttt{d}_2$ w.r.t. S if: Equivalence

$$\forall \mathfrak{I}, (\mathfrak{I} \models S) : \mathfrak{I}[\texttt{d}_1] = \mathfrak{I}[\texttt{d}_2]$$

Disjointness A concept $\texttt{d}_1$ is disjoint to $\texttt{d}_2$ w.r.t. S if: Disjointness

$$\forall \mathfrak{I}, (\mathfrak{I} \models S) : \mathfrak{I}[\texttt{d}_1] \neq \mathfrak{I}[\texttt{d}_2]$$

Theorem 3.3.1 (Reduction to subsumption)**.** Given the concepts $\texttt{d}_1$ and $\texttt{d}_2$, it holds that: Reduction to subsumption

- $\texttt{d}_1$ is unsatisfiable $\iff$ $\texttt{d}_1 \sqsubseteq \bot$.
- $\texttt{d}_1 \doteq \texttt{d}_2 \iff \texttt{d}_1 \sqsubseteq \texttt{d}_2 \land \texttt{d}_2 \sqsubseteq \texttt{d}_1$.
- $\texttt{d}_1$ and $\texttt{d}_2$ are disjoint $\iff$ $(\texttt{d}_1 \cap \texttt{d}_2) \sqsubseteq \bot$.

3.3.2 A-box reasoning

Given a constant $\texttt{c}$, a concept $\texttt{d}$ and a set of sentences S, we can determine the following:

Satisfiability A constant $\texttt{c}$ satisfies the concept $\texttt{d}$ if: Satisfiability

$$S \models (\texttt{c} \rightarrow \texttt{d})$$

Note that it can be reduced to subsumption.

3.3.3 Computing subsumptions

Given a knowledge base KB and two concepts `d` and `e`, we want to prove:

$$KB \models (\mathtt{d} \sqsubseteq \mathtt{e})$$

The following algorithms can be employed:

Structural matching

Structural matching

1. Normalize `d` and `e` into a conjunctive form:

$$\mathtt{d} = \mathtt{[AND\ d_1\ \dots d_n]} \qquad \mathtt{e} = \mathtt{[AND\ e_1\ \dots e_m]}$$

2. Check if each part of `e` is accounted by at least a component of `d`.

Tableaux-based algorithms

Tableaux-based algorithms Exploit the following theorem:

$$(KB \models (C \sqsubseteq D)) \iff (KB \cup (x : C \sqcap \neg D)) \text{ is inconsistent}$$

Note: similar to refutation.

3.3.4 Open world assumption

Open world assumption

Open world assumption If a sentence cannot be inferred, its truth values is unknown.

Description logics are based on the open world assumption. To reason in open world assumption, all the possible models are split upon encountering an unknown facts depending on the possible cases (Oedipus example).

3.4 Expanding description logic

It is possible to expand a description logic by:

Adding concept-forming operators

Adding concept-forming operators Let `r` be a role, `d` be a concept, `c` be a constant and n a positive integer. We can extend our description logic with:

`[AT-MOST` n `r]` Individuals `r`-related to at most n other individuals.

Example. `[AT-MOST 1 :Child]` individuals with only a child.

`[ONE-OF` $\mathtt{c}_1$ ... $\mathtt{c}_n$`]` Concept only satisfied by $\mathtt{c}_1$ ... $\mathtt{c}_n$.

Example. `Beatles` $\doteq$ `[ALL :BandMember [ONE-OF john paul george ringo]]`

`[EXISTS` n `r d]` Individuals `r`-related to at least n individuals in the category `d`.

Example. `[EXISTS 2 :Child Male]` individuals with at least two male children.

Note: this increases the computational complexity of entailment.

Relating roles

Relating roles

`[SAME-AS` $\mathtt{r}_1$ $\mathtt{r}_2$`]` Equates fillers of the roles r_1 and r_2

Example. `[SAME-AS :CEO :Owner]`

Note: this increases the computational complexity of entailment. Role chaining also leads to undecidability.

Adding rules

Adding rules Rules are useful to add conditions (e.g. `if` $\mathtt{d}_1$ `then [FILLS r c]`).

3.5 Description logics family

Depending on the number of operators, a description logic can be:

- More expressive.
- Computationally more expensive.
- Undecidable.

Attributive language ($\mathcal{AL}$) Minimal description logic with:

- Atomic concepts.
- Universal concept (`Thing` or $\top$).
- Bottom concept (`Nothing` or $\bot$).
- Atomic negation (only for atomic concepts).
- `AND` operator ($\sqcap$).
- `ALL` operator ($\forall$).
- `[EXISTS 1 r]` operator ($\exists$).

Attributive language complement ($\mathcal{ALC}$) $\mathcal{AL}$ with negation for concepts.

$\mathcal{F}$	Functional properties
$\mathcal{E}$	Full existential quantification
$\mathcal{U}$	Concept union
$\mathcal{C}$	Complex concept negation
$\mathcal{S}$	$\mathcal{ALC}$ with transitive roles
$\mathcal{H}$	Role hierarchy
$\mathcal{R}$	Limited complex roles axioms Reflexivity and irreflexivity Roles disjointness
$\mathcal{O}$	Nominals
$\mathcal{I}$	Inverse properties
$\mathcal{N}$	Cardinality restrictions
$\mathcal{Q}$	Qualified cardinality restrictions
$(\mathcal{D})$	Datatype properties, data values and data types

Table 3.1: Name and expressivity of logics

4 Web reasoning

4.1 Semantic web

Semantic web Method to represent and reason on the data available on the web. Semantic web aims to preserve the characteristics of the web, this includes: Semantic web

- Globality.
- Information distribution.
- Information inconsistency of contents and links (as everyone can publish).
- Information incompleteness of contents and links.

Information is structured using ontologies and logic is used as inference mechanism. New knowledge can be derived through proofs.

Uniform resource identifier Naming system to uniquely identify concepts. Each URI correspond to one and only one concept, but multiple URIs can refer to the same concept. URI

XML Markup language to represent hierarchically structured data. An XML can contain in its preamble the description of the grammar used within the document. XML

Resource description framework (RDF) XML-based language to represent knowledge. Based on triplets: Resource description framework (RDF)

```
<subject, predicate, object>
<resource, attribute, value>
```

RDF supports:

Types Using the attribute `type` which can assume an URI as value.

Collections Subjects and objects can be bags, sequences or alternatives.

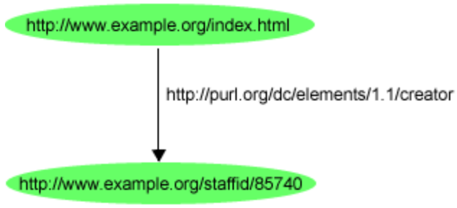
Meta-sentences Reification of the sentences (e.g. "X says that Y...").

RDF schema RDF can be used to describe classes and relations with other classes (e.g. `type`, `subClassOf`, `subPropertyOf`, ...) RDF schema

Representation

Graph A graph where nodes are subjects or objects and edges are predicates.

Example.



The graph stands for: `http://www.example.org/index.html` has a `creator` with staff id `85740`.

XML

Example.

```
<rdf:RDF
xmlns:rdf=http://www.w3.org/1999/02/22-rdf-syntax-ns#
xmlns:contact=http://www.w3.org/2000/10/swap/pim/contact#>
    <contact:Person rdf:about="http://www.w3.org/People/EM/
    contact#me">
        <contact:fullName>Eric Miller</contact:fullName>
        <contact:mailbox rdf:resource="mailto:em@w3.org"/>
        <contact:personalTitle>Dr.</contact:personalTitle>
    </contact:Person>
</rdf:RDF>
```

Database similarities RDF aims to integrate different databases:

- A DB record is a RDF node.
- The name of a column can be seen as a property type.
- The value of a field corresponds to the value of a property.

RDFa Specification to integrate XHTML and RDF. RDFa

SPARQL Language to query different data sources that support RDF (natively or through a middleware). SPARQL

Ontology web language (OWL) Ontology based on RDF and description logic fragments. Three level of expressivity are available: Ontology web language (OWL)

- OWL lite.
- OWL DL.
- OWL full.

An OWL has:

Classes Categories.

Properties Roles and relations.

Instances Individuals.

4.2 Knowledge graphs

Knowledge graph Knowledge graphs overcome the computational complexity of T-box reasoning with semantic web and description logics. Knowledge graph

- Use a simple vocabulary with a simple but robust corpus of types and properties adopted as a standard.
- Represent a graph with terms as nodes and edges connecting them. Knowledge is therefore represented as triplets `(h, r, t)` where `h` and `t` are entities and `r` is a relation.
- Logic formulas are removed. T-box and A-box can be seen as the same concept. There is no reasoning but only facts.

- Data does not have a conceptual schema and can come from different sources with different semantics.
- Graph algorithms to traverse the graph and solve queries.

KG quality Quality

Coverage If the graph has all the required information.

Correctness If the information is correct (can be objective or subjective).

Freshness If the content is up-to-date.

Graph embedding Project entities and relations into a vectorial space for ML applications. Graph embedding

Entity prediction Given two entities `h` and `t`, determine the relation `r` between them.

Link prediction Given an entity `h` and a relation `t`, determine an entity `t` related to `h`.

5 Quantitative time reasoning

5.1 Propositional logic

State The current state of the world can be represented as a set of propositions that are true according the observation of an agent. State

The union of a countable sequence of states represents the evolution of the world. Each proposition is distinguished by its time step.

Example. A child has a bow and an arrow, then shoots the arrow.

$$\text{KB}^0 = \{\texttt{hasBow}^0, \texttt{hasArrow}^0\}$$
$$\text{KB}^1 = \{\texttt{hasBow}^0, \texttt{hasArrow}^0, \texttt{hasBow}^1, \neg\texttt{hasArrow}^1\}$$

Action An action indicates how a state evolves into the next one. It is described using effect axioms in the form: Action

$$\texttt{action}^t \Rightarrow (\texttt{preconditions}^t \iff \texttt{effects}^{t+1})$$

Frame problem The effect axioms of an action do not tell what remains unchanged in the next state. Frame problem

Frame axioms The frame axioms of an action describe the unaffected propositions of an action. Frame axioms

Example. The action of shooting an arrow can be described as:

$$\texttt{SHOOT}^t \Rightarrow \{\texttt{hasArrow}^t \iff \neg\texttt{hasArrow}^{t+1}\}$$
$$\texttt{SHOOT}^t \Rightarrow \{\texttt{hasBow}^t \iff \texttt{hasBow}^{t+1}\}$$

Note that with m actions and n propositions, the number of frame axioms will be of order $O(mn)$. Inference for t time steps will have complexity $O(nt)$.

5.2 Situation calculus (Green's formulation)

Situation calculus uses first order logic instead of propositional logic.

Situation The initial state is a situation. Applying an action in a situation is a situation: Situation

$$s \text{ is a situation and } \texttt{a} \text{ is an action} \iff \texttt{result}(\texttt{a}, s) \text{ is situation}$$

(Note: in FAIRK module 1, `result` is denoted as `do`).

Fluent Function that varies depending on the situation (i.e. tells if a property holds in a given situation). Fluent

Example. $\texttt{hasBow}(s)$ where s is a situation.

Action Actions are described using:

Action

Possibility axioms Indicates the preconditions $\phi_{\texttt{a}}$ of an action $\texttt{a}$ in a given situation s:

Possibility axioms

$$\phi_{\texttt{a}}(s) \Rightarrow \texttt{poss}(\texttt{a}, s)$$

Successor state axiom The evolution of a fluent F follows the axiom:

Successor state axiom

$$F^{t+1} \iff (\texttt{ActionCauses}(F) \vee (F^t \wedge \neg\texttt{ActionCauses}(\neg F)))$$

In other words, a fluent is true if an action makes it true or does not change if the action does not involve it.

Adding the notion of possibility, an action can be described as:

$$\begin{aligned}\texttt{poss}(\texttt{a}, s) \Rightarrow \Big(F(\texttt{result}(\texttt{a}, s)) \iff\\ (\texttt{a} = \texttt{ActionCauses}(F)) \vee\\ (F(s) \wedge \texttt{a} \neq \neg\texttt{ActionCauses}(\neg F))\Big)\end{aligned}$$

Unique action axiom Only a single action can be executed in a situation to avoid non-determinism.

Unique action axiom

5.3 Event calculus (Kowalski's formulation)

Event calculus reifies fluents and events (actions) as terms (instead of predicates).

Event calculus ontology A fixed set of predicates:

Event calculus ontology

$\texttt{holdsAt}(F, T)$ The fluent F holds at time T.

$\texttt{happens}(\texttt{E}, T)$ The event $\texttt{E}$ (i.e. execution of an action) happened at time T.

$\texttt{initiates}(\texttt{E}, F, T)$ The event $\texttt{E}$ causes the fluent F to start holding at time T.

$\texttt{terminates}(\texttt{E}, F, T)$ The event $\texttt{E}$ causes the fluent F to cease holding at time T.

$\texttt{clipped}(T_i, F, T_j)$ The fluent F has been made false between the times T_i and T_j ($T_i < T_j$).

$\texttt{initially}(F)$ The fluent F holds at time 0.

Domain-independent axioms A fixed set of axioms:

Domain-independent axioms

Truthness of a fluent

1. A fluent holds if an event initiated it in the past and has not been clipped.

$$\begin{aligned}\texttt{holdsAt}(F, T_j) \Leftarrow & \texttt{happens}(\texttt{E}, T_i) \wedge (T_i < T_j) \wedge\\ & \texttt{initiates}(\texttt{E}, F, T_i) \wedge \neg\texttt{clipped}(T_i, F, T_j)\end{aligned}$$

2. A fluent holds if it was initially true and has not been clipped.

$$\texttt{holdsAt}(F, T) \Leftarrow \texttt{initially}(F) \wedge \neg\texttt{clipped}(0, F, T)$$

Note: the negations make the definition of these axioms in Prolog unsafe.

Clipping of a fluent

$$\texttt{clipped}(T_i, F, T_j) \Leftarrow \texttt{happens}(\texttt{E}, T) \land (T_i < T < T_j) \land \texttt{terminates}(\texttt{E}, F, T)$$

Domain-dependent axioms Domain-specific axioms defined using the predicates `initially`, `initiates` and `terminates`.

Domain-dependent axioms

Deductive reasoning Event calculus only allows deductive reasoning: it takes as input the domain-dependant axioms and a set of events, and computes a set of true fluents. If a new event is observed, the query need to be recomputed again.

Example. A room with a light and a button can be described as:

Fluents `lightOn` · `lightOff`

Events `PUSH_BUTTON`

Domain-dependent axioms are:

Initial state `initially(lightOff)`

Effects of `PUSH_BUTTON` on `lightOn`

- $\texttt{initiates}(\texttt{PUSH\_BUTTON}, \texttt{lightOn}, T) \Leftarrow \texttt{holdsAt}(\texttt{lightOff}, T)$
- $\texttt{terminates}(\texttt{PUSH\_BUTTON}, \texttt{lightOn}, T) \Leftarrow \texttt{holdsAt}(\texttt{lightOn}, T)$

Effects of `PUSH_BUTTON` on `lightOff`

- $\texttt{initiates}(\texttt{PUSH\_BUTTON}, \texttt{lightOff}, T) \Leftarrow \texttt{holdsAt}(\texttt{lightOn}, T)$
- $\texttt{terminates}(\texttt{PUSH\_BUTTON}, \texttt{lightOff}, T) \Leftarrow \texttt{holdsAt}(\texttt{lightOff}, T)$

A set of events could be:

$$\texttt{happens}(\texttt{PUSH\_BUTTON}, 3) \cdot \texttt{happens}(\texttt{PUSH\_BUTTON}, 5) \cdot \texttt{happens}(\texttt{PUSH\_BUTTON}, 6)$$

5.3.1 Reactive event calculus

Allows to add events dynamically without the need to recompute the result.

Reactive event calculus

5.4 Allen's logic of intervals

Event calculus only captures instantaneous events that happen in given points in time.

Allen's logic of intervals Reasoning on time intervals.

Allen's logic of intervals

Interval An interval i starts at a time $\texttt{begin}(i)$ and ends at a time $\texttt{end}(i)$.

Interval

Temporal operators

Temporal operators

- $\texttt{meet}(i, j) \iff \texttt{end}(i) = \texttt{begin}(j)$
- $\texttt{before}(i, j) \iff \texttt{end}(i) < \texttt{begin}(j)$
- $\texttt{after}(i, j) \iff \texttt{before}(j, i)$
- $\texttt{during}(i, j) \iff \texttt{begin}(j) < \texttt{begin}(i) < \texttt{end}(i) < \texttt{end}(j)$
- $\texttt{overlap}(i, j) \iff \texttt{begin}(i) < \texttt{begin}(j) < \texttt{end}(i) < \texttt{end}(j)$

- $\texttt{starts}(i, j) \iff \texttt{begin}(i) = \texttt{begin}(j)$
- $\texttt{finishes}(i, j) \iff \texttt{end}(i) = \texttt{end}(j)$
- $\texttt{equals}(i, j) \iff \texttt{starts}(i, j) \land \texttt{ends}(i, j)$

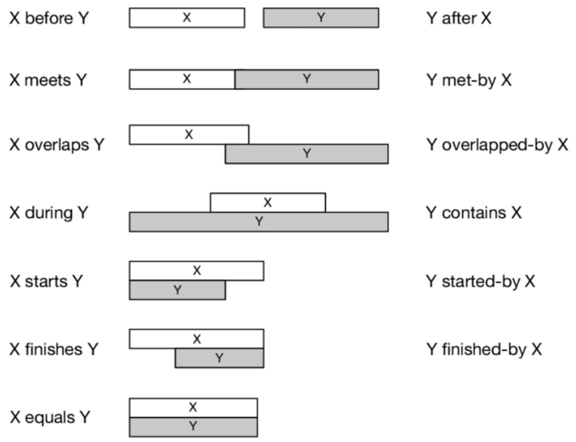


Figure 5.1: Visual representation of temporal operators