Fundamentals of Artificial Intelligence and Knowledge Representation (Module 3)

Last update: 13 October 2023

Academic Year 2023 – 2024

Alma Mater Studiorum · University of Bologna

Contents

1 Introduction

1.1 Uncertainty

Uncertainty A task is uncertain if we have: Uncertainty

- Partial observations
- Noisy or wrong information
- Uncertain action outcomes
- Complex models

A purely logic approach leads to:

- Risks falsehood: unreasonable conclusion when applied in practice.
- Weak decisions: too many conditions required to make a conclusion.

1.1.1 Handling uncertainty

Default/nonmonotonic logic Works on assumptions. An assumption can be contradicted by an evidence. Default/nonmonotonic logic

Rule-based systems with fudge factors Formulated as premise $\rightarrow_{\text{prob.}}$ effect. Have the following issues: Rule-based systems with fudge factors

- Locality: how can the probability account all the evidence.
- Combination: chaining of unrelated concepts.

Probability Assign a probability given the available known evidence. Probability

Note: fuzzy logic handles the degree of truth and not the uncertainty.

Decision theory Defined as: Decision theory

$$\text{Decision theory} = \text{Utility theory} + \text{Probability theory}$$

where the utility theory depends on one's preferences.

1.1.2 Probability

Sample space Set Ω of all possible worlds. Sample space

Event Subset $A \subseteq \Omega$. Event

Sample point/Possible world/Atomic event Element $\omega \in \Omega$. Sample point

Probability space A probability space/model is a function $\mathcal{P}(\cdot) : \Omega \rightarrow [0, 1]$ assigned to a sample space such that: Probability space

- $0 \leq \mathcal{P}(\omega) \leq 1$
- $\sum_{\omega \in \Omega} \mathcal{P}(\omega) = 1$

- $\mathcal{P}(A) = \sum_{\omega \in A} \mathcal{P}(\omega)$

Random variable A function from an event to some range (e.g. reals, booleans, ...). Random variable

Probability distribution For any random variable X: Probability distribution

$$\mathcal{P}(X = x_i) = \sum_{\omega \text{ st } X(\omega)=x_i} \mathcal{P}(\omega)$$

Proposition Event where a random variable has a certain value. Proposition

$$a = \{\omega \mid A(\omega) = \texttt{true}\}$$

$$\neg a = \{\omega \mid A(\omega) = \texttt{false}\}$$

$$(\texttt{Weather} = \texttt{rain}) = \{\omega \mid B(\omega) = \texttt{rain}\}$$

Prior probability Prior/unconditional probability of a proposition based on known evidence. Prior probability

Probability distribution (all) Gives all the probabilities of a random variable. Probability distribution (all)

$$\mathbf{P}(A) = \langle \mathcal{P}(A = a_1), \dots, \mathcal{P}(A = a_n) \rangle$$

Joint probability distribution The joint probability distribution of a set of random variables gives the probability of all the different combinations of their atomic events. Joint probability distribution

Note: Every question on a domain can, in theory, be answered using the joint distribution. In practice, it is hard to apply.

Example. $\mathbf{P}(\texttt{Weather}, \texttt{Cavity}) =$

	`Weather=sunny`	`Weather=rain`	`Weather=cloudy`	`Weather=snow`
`Cavity=true`	0.144	0.02	0.016	0.02
`Cavity=false`	0.576	0.08	0.064	0.08

Probability density function The probability density function (PDF) of a random variable X is a function $p : \mathbb{R} \to \mathbb{R}$ such that: Probability density function

$$\int_{\mathcal{T}_X} p(x)\, dx = 1$$

Uniform distribution Uniform distribution

$$p(x) = \text{Unif}[a, b](x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Gaussian (normal) distribution Gaussian (normal) distribution

$$\mathcal{N}(\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$\mathcal{N}(0, 1)$ is the standard gaussian.

Conditional probability Probability of a prior knowledge with new evidence: Conditional probability

$$\mathcal{P}(a|b) = \frac{\mathcal{P}(a \wedge b)}{\mathcal{P}(b)}$$